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Balancing Transport Efficiency and Bilateral Exchange Debts in Pallet Pooling

Niklas Jost [*1], *Dominik Sparer* [2], *Alexander Gehl* [1], *Leon Krebs* [2], *Jérôme Rutinowski* [1], *Alice Kirchheim* [1,2]

[1] Chair of Material Handling and Warehousing, TU Dortmund University, 44227 Dortmund, Germany
[2] Fraunhofer Institute for Material Flow and Logistics, 44227 Dortmund, Germany

Pallet pooling systems enable the shared use of returnable transport items across multiple independent clients. A central operational challenge in such systems is the joint management of physical pallet flows and persistent bilateral imbalances arising from repeated exchanges between small companies. In this work, we address this challenge by introducing the *Pallet Pooling Problem*, a mathematical formulation that simultaneously minimizes total transport distance and deviations from existing bilateral pallet debts. Recognizing that operational priorities differ across industries, we discuss alternative objective functions that provide a flexible decision-support framework. We further present an open-source, optimal solution approach for the Pallet Pooling Problem, which scales to large instances and is easily adaptable. The approach is validated on operational data of a German SME.

[Keywords: Pallet Pooling, Bilateral Exchange Debts, Network Flow Optimization]

1 Introduction

Pooling services for pallets and other returnable transport items constitute an essential component of collaborative logistics networks. Within such systems, participating companies typically encounter heterogeneous pallet situations: while some temporarily hold inventory surpluses, others face shortages that must be compensated through redistribution. Ensuring pallet availability is therefore a recurring operational task and a prerequisite for stable production and distribution processes.

The resolution of these imbalances is inherently linked to physical transportation. Pallet relocation require physical transportation, which introduces distance-dependent costs and operational effort. Consequently, pooling service providers face the recurring challenge of coordinating pallet exchanges across geographically dispersed partners in a cost-efficient manner. At the same time, these exchanges are embedded in long-term business relationships, in which repeated interactions shape expectations, responsibilities, and implicit obligations among participants.

A distinctive characteristic of real-world pooling systems is the accumulation of bilateral exchange imbalances over time. When pallets are supplied without immediate compensation, implicit debts arise between specific partners. These debts are typically recorded for accounting or operational purposes and influence future cooperation. While they do not directly affect physical feasibility, they play an important role in maintaining balanced and sustainable relationships within the network. Routing decisions that repeatedly favor certain partners may be perceived as unfair or strategically undesirable, even if they are efficient from a purely logistical perspective.

Existing operational decision-making often treats these two dimensions separately. Physical redistribution focuses on minimizing transport effort and satisfying demand, whereas debt tracking is handled administratively without direct feedback into routing decisions. This separation obscures an important interface between distance minimization and relational balance: operational transport decisions influence the evolution of bilateral debts, and existing debts, in turn, shape expectations regarding future exchanges.

This paper addresses this gap by studying pallet pooling in a setting where physical transport efficiency and the maintenance of balanced bilateral exchange relations are jointly relevant. We make the following contributions:

- We introduce a mathematical formulation of the Pallet Pooling Problem that explicitly integrates transport costs and bilateral pallet debts.
- We provide a time-efficient, optimal, and open-source solution capable of handling large-scale instances.

- We discuss alternative objective functions and extensions that reflect different operational priorities.

The remainder of this paper is structured as follows. Section 2 reviews related work on pallet pooling, asset repositioning, and debt-clearing mechanisms. Section 3 introduces a mathematical formulation that integrates physical pallet movements with bilateral exchange relations and discusses solution approaches. Section 4 examines alternative objective structures and possible extensions. Section 5 concludes the paper and outlines directions for future research.

2 Related Work

The literature relevant to this work originates from three areas of research: physical pooling and returnable transport item (RTI) management, the repositioning of empty assets such as shipping containers, and debt-clearing models from financial network theory.

Research on pallets and RTIs has examined circulation processes, inventory levels, routing, and closed-loop flows within collaborative logistics networks [1, 2]. It is evident that such models characteristically seek to stabilize surpluses and deficits by means of redistribution, frequently resorting to inventory-routing or flow-based formulations in the process [2, 3]. These models align with the structural logic of the problem by addressing physical flows across multiple actors. However, these models do not take into account exchange imbalances between partners or the accumulation of debt, instead treating the system exclusively through aggregate stock levels and operational efficiency [1].

A secondary pertinent research trajectory pertains to the global repositioning of empty containers within the domain of maritime logistics. The phenomenon of surpluses and shortages in container stocks bears a striking resemblance to the dynamics of pallet imbalances [4, 5]. Numerous studies have sought to address this issue by formulating it as a minimum-cost flow problem [4]. These approaches emphasize the minimization of transport costs, the satisfaction of demand, and the deployment of fleets [5, 6]. While there is some similarity in the underlying structure of these models, it should be noted that they do not incorporate bilateral obligations between locations. The treatment of imbalances is undertaken on an aggregated basis, as opposed to a linkage to specific partners [6].

The concept of mutual debts is instead explored in the context of financial network literature. In this context, nodes represent financial institutions connected through bilateral liabilities, typically represented by a liability matrix [7]. Clearing refers to the settlement of outstanding obligations under default conditions and can be computed as the solution of an optimization problem. The resulting models are formulated in an abstract financial network setting and focus on the settlement of monetary obligations [7].

To the best of our knowledge, no work has been identified that jointly considers combining physical transport flows with explicit bilateral debt relations for pooling services in a single optimization model. Existing pooling and repositioning models are characterized by an absence of a debt component [2, 5], while financial debt-clearing models are unconstrained by physical limitations. This work establishes a novel approach that jointly considers distance-based transport efficiency and the preservation of bilateral pallet debts, thereby addressing a gap in the literature.

3 Mathematical Formulation

This section presents the mathematical formulation of the pallet pooling problem.

We model this problem as a complete graph problem with $G = (V, E)$. Let V denote the set of clients, each associated with a net pallet balance $r_v \in \mathbb{Z}$ for all $v \in V$. A negative value r_v indicates that client v requires $|r_v|$ pallets, while a positive value indicates a surplus of r_v pallets. Moreover, the parameter $p_{ij} \geq 0$ gives the number of pallets client i owes client j. We might assume $p_{ij} = 0$ or $p_{ji} = 0$. The distance between two clients $i, j \in V$ is given by $d_{ij} \geq 0$ with $(i, j) \in E$. The goal is to determine a number of pallets x_{ij} transported between any pair of clients i, j satisfying all pallet demands. We minimize both the total transport cost and the summed bilateral debts and scale the latter by $\mu \geq 0$. The model parameters are summarized in Table 1.

Table 1: Model parameters and decision variables.

Symbol	Type	Description
V	Set	Set of clients (nodes in the network)
r_i	Parameter	Net pallet balance of client i ($r_i < 0$: demand; $r_i > 0$: surplus)
d_{ij}	Parameter	Distance between client i and client j
p_{ij}	Parameter	Debt of client i towards client j
μ	Parameter	Scaling factor balancing transport cost and debt deviation
x_{ij}	Variable	Number of pallets transported from client i to client j

The problem reads:

$$\min \quad \sum_{i\in V}\sum_{j\in V} d_{ij}x_{ij} + \mu \sum_{i\in V}\sum_{j\in V} |p_{ij} - x_{ij}| \quad (1)$$

$$\text{s.t.} \quad \sum_{j\in V} x_{ji} - \sum_{j\in V} x_{ij} + r_i \geq 0 \quad \forall i \in V \quad (2)$$

$$x_{ij} \geq 0 \quad \forall i,j \in V \quad (3)$$

The objective function (1) minimizes a weighted combination of total transport distance and the summed bilateral debts respecting past debts. The first term represents the total transport cost, while the second term penalizes the difference between the realized pallet flows x_{ij} and the pre-existing debts p_{ij}. The trade-off between both components is controlled by the scaling parameter μ.

Constraint (2) ensures that the final pallet balance of each client is non-negative, i.e., each client's demand is satisfied by incoming flows and their own surplus. Constraint (3) enforces non-negativity of pallet flows.

We assume that the total number of pallets in the system is sufficient, i.e., $\sum_{i\in V} r_i \geq 0$. If $\sum_{i\in V} r_i < 0$, all signs can be reversed to obtain an equivalent formulation where any pallet must be in use to provide a maximal utilization. Alternatively, a dummy node with $r_0 = -\sum_{i\in V} r_i$ can be added to balance the system.

Two approaches were implemented to solve the model efficiently:

The first approach is devided into two steps. In the first step, the algorithm minimizes total debt deviation using a flow network, where nodes with $r_i > 0$ act as sources and nodes with $r_j < 0$ as sinks. Edges were added for every $p_{ij} > 0$. Standard flow algorithms are then applied to minimize $\sum_i |r_i|$. In a second step, the flow is recomputed without considering debts but distances instead, to ensure that all demands are satisfied. Although intuitive, this procedure proved computationally demanding in practice.

To improve efficiency, as a second approach the problem was solved using an optimal matching algorithm. This approach handles up to 10,000 nodes within a few seconds on a standard computer, demonstrating significantly improved scalability compared to the heuristic. The code is open source and publicly available via Zenodo (DOI: 10.5281/zenodo.18300159) and is referenced in [8]. It was tested on real world operational data and yields optimal results, consistent to desirable exchanges suggested in an expert interview.

The proposed formulation captures the trade-off between minimizing transport distances and maintaining fair debt relations among clients. In practice, there is no universally accepted objective, as clients may value distance minimization and debt balancing differently. Alternative objectives and extensions are discussed in Section 4, which can easily be integrated in the existing code.

4 Discussion

The formulation in Section 3 highlights an inherent tension between minimizing transport distances and maintaining balanced bilateral debt relations across clients. The relative importance of these two components is application-specific and may differ across industries, contractual settings, or organizational policies. The linear objective in (1) represents one plausible compromise: deviations from historical debts are penalized proportionally, implying that a debt of twice the magnitude is considered twice as undesirable. While this assumption provides interpretability and computational convenience, it may not fully capture operational or behavioral priorities observed in practice.

A first extension concerns the treatment of debt magnitude. In some settings, a single large outstanding debt may be regarded as particularly problematic as it presents a concentration risk or imbalanced business relationships. Conversely, distributing obligations across multiple partners may be viewed more favorably, analogous to reducing variance in a portfolio. These considerations suggest replacing the linear deviation penalty with a quadratic term, allowing the model to discourage large imbalances more aggressively:

$$\min \sum_{i\in V}\sum_{j\in V} d_{ij}x_{ij} \; + \; \mu \sum_{i\in V}\sum_{j\in V} (p_{ij} - x_{ij})^2.$$

A second extension accounts for the duration of outstanding debts. In many operational contexts, long-standing obligations are prioritized for resolution, either to maintain trust among partners or to comply with internal financial policies. To reflect this, we introduce a parameter $t_{ij} \geq 0$ representing the age of the debt from client i to client j, which is reset to zero whenever the corresponding obligation is cleared. Incorporating t_{ij} as a weighting factor yields

$$\min \sum_{i\in V}\sum_{j\in V} d_{ij}x_{ij} \; + \; \mu \sum_{i\in V}\sum_{j\in V} t_{ij}\,|p_{ij} - x_{ij}|,$$

which increases the incentive to resolve older debts while retaining the interpretability of the linear absolute deviation term. Depending on operational preferences, the aging process for t_{ij} may follow a simple linear update, a capped growth rule, or a cyclic scheme analogous to round-robin prioritization.

Naturally, these extensions can also be combined. For example, age-based weights may be applied together with quadratic penalties, or different classes of clients may employ distinct weighting schemes. The modularity of the formulation allows such alternative objectives and refinements to be integrated directly into the existing solution framework without altering the underlying flow structure of the model.

5 Conclusion

This paper introduced a mathematical formulation for pallet pooling that integrates two dimensions which, to date, have been treated separately in the literature: the optimization of physical transport flows and the preservation of bilateral pallet debts among clients. By combining these elements into a unified objective, the model captures a more realistic representation of operational practice, wherein both logistical efficiency and fairness in reciprocal exchanges play a central role.

The proposed formulation offers several advantages. First, it provides a flexible framework that allows users to specify the relative importance of transport minimization and debt balancing. This reflects the heterogeneity of priorities across industries and operational settings. Second, the model structure remains computationally tractable. While heuristic approaches based purely on flow decomposition exhibited scalability limitations, the matching implementation demonstrated the ability to solve instances with up to 10,000 nodes within seconds on standard hardware. This makes the approach suitable for real-time or large-scale decision support. Third, the model is extensible: quadratic penalties, debt-aging mechanisms, and hybrid weighting schemes can be incorporated without altering the underlying flow constraints, thereby enabling tailored configurations for different use cases.

The work also highlights opportunities for further research. One intuitive direction is the investigation of dynamic or multi-period variants in which pallet flows and debts evolve over time. Integrating stochastic demand or uncertainty in pallet returns may improve applicability in volatile environments. In addition, the incorporation of strategic considerations as contractual incentives, partner reliability, or credit limits could enrich the behavioral realism of the framework. Another extension concerns heterogeneity in pallet quality. In practice, exchanged pallets may differ with respect to condition, wear, or compliance with quality standards, which can influence both acceptance and perceived fairness of exchanges. Explicitly accounting for quality differences, for instance through quality-dependent weights or constraints, could further enhance the realism of the model. Finally, extensive empirical validation using different industries, datasets, and operational settings would help establish broader generalizability.

Taken together, this study provides the first optimization model that jointly addresses physical pallet movements and bilateral debt relations. By bridging the gap between pooling logistics and financial network concepts, it advances both theoretical understanding and practical decision-making in collaborative pallet systems.

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Niklas Jost, M.Sc.*, Chair of Material Handling and Warehousing (FLW), TU Dortmund University.
Alexander Gehl, B.Sc., Chair of Material Handling and Warehousing (FLW), TU Dortmund University.
Dipl.-Inf. Dominik Sparer, Fraunhofer Institute for Material Flow and Logistics IML.
Dr.-Ing. Jérôme Rutinowski, Chair of Material Handling and Warehousing (FLW), TU Dortmund University. **Leon Krebs, M.Sc.**, Fraunhofer Institute for Material Flow and Logistics IML.
Prof. Dr.-Ing. Alice Kirchheim, Fraunhofer Institute for Material Flow and Logistics IML, Chair of Material Handling and Warehousing (FLW), TU Dortmund University.

Address: Chair of Material Handling and Warehousing (FLW), LogistikCampus, Joseph-von-Fraunhofer-Str. 2-4, 44227 Dortmund, Germany.

Address: Fraunhofer Institute for Material Flow and Logistics IML, Joseph-von-Fraunhofer-Straße 2-4, 44227 Dortmund, Germany.

*Corresponding author: niklas.jost@tu-dortmund.de